

Pre-Calculus Review 2.1-2.3 Show all work!

Formulas you may need for this test:

$y = a(x - h)^2 + k$

$y = ax^2 + bx + c$

vertex (h, k)

$h = \frac{-b}{2a}$

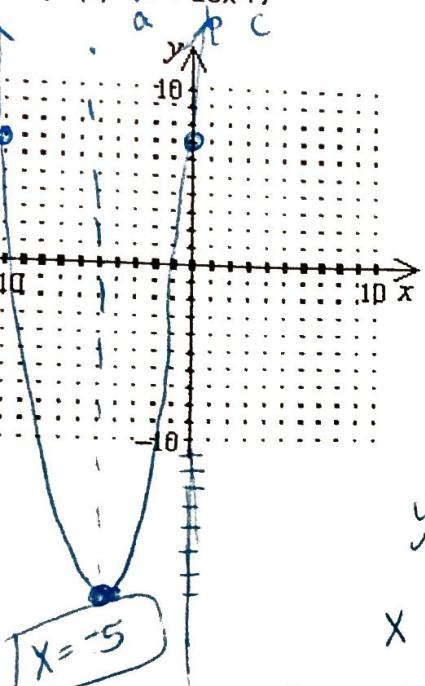
$k = f\left(\frac{-b}{2a}\right)$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Find the vertex form of the equation, find the vertex, equation of axis of symmetry, and x& y-intercepts.

Does it have a max or min and if so which and give the order pair. Graph

a) $f(x) = x^2 + 10x + 7$



$h = \frac{-b}{2a}$

$K = f\left(\frac{-b}{2a}\right)$

$h = \frac{-10}{2(1)} = -5$

$K = (-5)^2 + 10(-5) + 7$

$K = -18$

$y = 1(x + 5)^2 - 18$

Vertex $(-5, -18)$

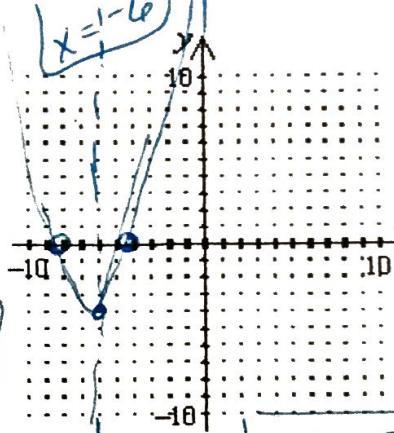
y int.

$y = (0)^2 + 10(0) + 7 = 7$

$x = \frac{-10 \pm \sqrt{100 - 4(1)(7)}}{2(1)}$

$= \frac{-10 \pm \sqrt{72}}{2} = \frac{-10 \pm 6\sqrt{2}}{2} = \boxed{\left(5 \pm 3\sqrt{2}, 0\right)}$

b) $f(x) = x^2 + 12x + 32$



$h = \frac{-12}{2(1)} = \frac{-12}{2} = -6$

$K = (-6)^2 + 12(-6) + 32 = 32$

vertex $\boxed{(-6, -4)}$

y int.

$y = (0)^2 + 12(0) + 32 = 32$

$\boxed{(0, 32)}$

$x = \frac{-12 \pm \sqrt{144 - 4(1)(32)}}{2(1)}$

$x = \frac{-12 \pm \sqrt{144 - 128}}{2} = \frac{-12 \pm \sqrt{16}}{2} = \frac{-12 \pm 4}{2} = \boxed{-8, -4}$

$x = \frac{-10 \pm \sqrt{72}}{2} = \frac{-10 \pm 6\sqrt{2}}{2} = \boxed{\left(5 \pm 3\sqrt{2}, 0\right)}$

Write the vertex form of the quadratic function if the parabola goes through the point $(7, 15)$ with a vertex at $(5, 12)$ remember to use: $Y = a(x-h)^2 + k$

$15 = a(7-5)^2 + 12$

$15 = a(2)^2 + 12$

$15 = 4a + 12$

$3 = 4a$

$a = \frac{3}{4}$

$f(x) = \frac{3}{4}(x-5)^2 + 12$

Write the simplest (degree 3) polynomial functions with zeros $-3, 5, 1$

$$(x+3)(x-5)(x-1)$$

$\underbrace{\hspace{1cm}}$ roots

$$(x^2 - 2x - 15)(x-1)$$

$$\begin{aligned} x = -3 &\Rightarrow (x+3) \text{ is factor} \\ x = 5 &\Rightarrow (x-5) \\ x = 1 &\Rightarrow (x-1) \end{aligned}$$

$$f(x) = x^3 - x^2 - 2x^2 + 2x - 15x + 15 = \boxed{x^3 - 3x^2 - 13x + 15 = f(x)}$$

SHOW WORK

Find the real zeros of the quadratic function using the quadratic formula.

a b c

a) $x^2 - 4x - 8 = 0$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)}$

$\frac{4 \pm 4\sqrt{3}}{2}$

$\boxed{2 \pm 2\sqrt{3}}$

b) $-3x^2 - 6x - 9 = -13$

$-3x^2 - 6x + 4 = 0$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(-3)(4)}}{2(-3)} = \frac{6 \pm \sqrt{48}}{2(-3)}$

$\boxed{-1}$

5. Find all the zeros of the polynomial and determine the multiplicity of each zero $f(x) = x^4 - 25x^2$

$$0 = x^2(x^2 - 25)$$

$$x^2 = 0$$

$$\boxed{x=0 \text{ (x2)}}$$

$$(x-5)(x+5)$$

$$\boxed{(x-5)=0}$$

$$\boxed{x=5 \text{ (x1)}}$$

$$\boxed{(x+5)=0}$$

$$\boxed{x=-5 \text{ (x1)}}$$

$$\boxed{-x^2=25x^2}$$

\uparrow even

negative

6. Describe the number of zeros, number of turns, and end behavior of the polynomial $f(x)$

4 zeros

3 turns

opens down to left & down to right

7. The value of Jennifer's stock portfolio is given by the function $v(t) = 50 + 73t - 3t^2$, where v is the value of the portfolio in hundreds of dollars and t is the time in months. How much money did Jennifer start with? When will the value of Jennifer's portfolio be at a maximum?

$$V(t) = -3t^2 + 73t + 50$$

$$V = -3\left(\frac{73}{6}\right)^2 + 73\left(\frac{73}{6}\right) + 50 = \frac{494}{6} \approx \$494 \text{ hundred}$$

Jennifer started with \$5000.00

$$h = \frac{-b}{2a} = \frac{-73}{2(-3)} = \frac{73}{6} \approx 12.2 \text{ months}$$

(12.2 months)

Divide the following problem using long division and synthetic.

$$8. \frac{x^4 + 2x^3 - 13x^2 - 28x + 24}{x+3}$$

$$\begin{array}{r} 1 \quad 2 \quad -13 \quad -28 \quad 24 \\ \downarrow \quad -3 \quad 3 \quad 30 \quad -6 \\ 1 \quad -1 \quad -10 \quad 2 \quad \boxed{18} \end{array} \quad x^3 - x^2 - 10x + 2$$

$$x+3 \overline{)x^4 + 2x^3 - 13x^2 - 28x + 24} \\ - (x^4 + 3x^3) \\ \underline{-x^3 - 13x^2} \\ - (-x^2 - 3x^2)$$

Divide using long division

$$9. \frac{x^4 + 5x^3 - 20x - 16}{x^2 - x - 3}$$

$$\begin{array}{r} 2x + 24 \\ \underline{- (2x + 6)} \\ \quad \quad \quad 18 \end{array}$$

$$x^2 + 6x + 9 + \frac{7x + 11}{x^2 - x - 3}$$

$$x^2 - x - 3 \overline{)x^4 + 5x^3 + 0x^2 - 20x - 16} \\ - (x^4 - x^3 - 3x^2)$$

$$\underline{- (6x^3 + 3x^2 - 20x)} \\ - (6x^3 - 6x^2 - 18x)$$

$$\begin{array}{r} 9x^2 - 2x - 16 \\ \underline{- (9x^2 - 9x - 27)} \\ \quad \quad \quad 7x + 11 \end{array}$$

10. Is $(x+2)$ a factor of $2x^4 - 5x^2 + 5x - 2$? (use long or synthetic division)

$$\begin{array}{r} 2 \quad 0 \quad -5 \quad 5 \quad -2 \\ \downarrow \quad -4 \quad 8 \quad -6 \quad 2 \\ 2 \quad -4 \quad 3 \quad -1 \quad \boxed{0} \end{array}$$

Yes!!!

no remainder

11. Use synthetic division to evaluate $f(2)$ given that $f(x) = 3x^3 - 19x^2 + 2x + 24$

$$2 \left| \begin{array}{cccc} 3 & -19 & 2 & 24 \\ 4 & 6 & -26 & -48 \\ \hline 3 & -13 & -24 & \end{array} \right. \quad \boxed{-24}$$

$$\checkmark \quad 3(2)^3 - 19(2)^2 + 2(2) + 24$$

$$3 \cdot 8 - 19 \cdot 4 + 4 + 24$$

$$24 - 76 + 4 + 24$$

$$-52 + 4 + 24$$

Old way is to
Plug in

12. Explain the end behavior or draw a sketch of each case below.

- a. Degree is even, leading coefficient is negative
 - b. Degree is odd, leading coefficient is negative
 - c. Degree is even, leading coefficient is positive
 - d. Degree is odd, leading coefficient is positive

case below.

- down left & down right 48 + 24
- up left, down right - 24
- up left, up right
- down left, up right

FOR 13 -17 use the polynomial below:

$$f = x^5 - 5x^3 + 4x$$

13. Use the leading coefficient test to determine the end behavior of the graph of the polynomial on the left down & right up

14. The # of possible zeros

15. The # of possible turns 4

$$y = \frac{x^5 - 5x^3 + 4x}{x(x^4 - 5x^2 + 4)}$$

$$\frac{x(x^2-4)(x^2-1)}{x(x+2)(x-2)(x+1)(x-1)}$$

- ### 16. Find the actual zeros(roots)by factoring.

17. Use the roots and the multiplicity of each to draw a sketch of the graph.

