Notes: 2.4 Complex/Imaginary Numbers

Remember: $i = \sqrt{-1}$ Imaginary unit

By adding real numbers to real multiples of this imaginary unit, the set of **complex numbers** is obtained.

Definition of a Complex Number

Let a and b be real numbers. The number $a + bi$ is called a **complex number,** and it is said to be written in **standard form.** The real number a is called the **real part** and the real number b is called the **imaginary part** of the complex number.

When $b = 0$, the number $a + bi$ is a real number. When $b \neq 0$, the number $a + bi$ is called an **imaginary number.** A number of the form bi, where $b \neq 0$, is called a **pure imaginary number.**

For instance, the <u>**standard form**</u> of the complex number $-5 + \sqrt{-9}$ is $\underline{-5 + 3i}$

Find the sum and difference of the following complex numbers

Ex. 1

a. $7 + 3i + 5 - 4i$ $(7+5) + (3-4)i = \boxed{12 - i}$

b. $3 + 4i - (5 - 3i)$ $(3-5) + (4+3)i = \boxed{-2 + 7i}$

$-5 + 3i$

c. $2i ~~+~~ -3 - 4i - (-3 - 3i)$

$(\cancel{-3+3}) + (2-4+3)i$

$\boxed{i}$

Multiplying with complex numbers:

* ***remember if $i = \sqrt{-1}$ then $i^2 = -1$***

a. $(2-4i)(3+3i)$

$6 + 6i - 12i - 12i^2$

$6 - 6i - 12i^2$

$6 - 6i - 12(-1)$

$6 - 6i + 12$

$\boxed{18 - 6i}$

b. $(4+5i)^2$

$(4+5i)(4+5i)$

$16 + 20i + 20i + 25i^2$

$16 + 40i + 25i^2$

$16 + 40i + 25(-1)$

$\boxed{-9 + 40i}$

$i = \sqrt{-1} = i$
$i^2 = -1$
$i^3 = -1\sqrt{-1} = -i$
$i^4 = 1$

c. $(3-2i)(3+2i)$

13

d. $(4-6i)(4+6i)$

52

$$\begin{array}{r} 90\text{ r }2 \\ 4\overline{)362} \\ -36\downarrow \\ \hline 02 \end{array}$$

***Ex. c and d show you how two complex numbers can produce a real number, when multiplying. These are examples of multiplying **conjugates**.

Quotients of Complex numbers " keep it real"

The quotient of two complex numbers is in standard form, after the numerator and denominator have been <u>multiplied by the conjugate</u> to end with a <u>real number as the denominator</u>.

Ex. 3 Write in standard form

$$\frac{2+3i}{4-2i}\cdot\frac{(4+2i)}{(4+2i)} = \frac{8+4i+12i+6i^2}{16-\cancel{8i}+\cancel{8i}-4i^2}$$

$$\frac{8+16i-6}{16+4} = \frac{2+16i}{20} = \frac{1+8i}{10}$$

$$\boxed{\frac{1}{10}+\frac{4}{5}i}$$

Ex. 4 Write in standard form

$$\frac{2+i}{2-i}\cdot\frac{(2+i)}{(2+i)} = \frac{4+2i+2i+i^2}{4+2i-2i-i^2} = \frac{4+4i-1}{4-(-1)}$$

$$= \frac{3+4i}{5} = \boxed{\frac{3}{5}+\frac{4}{5}i}$$

Principal Square Root of a Negative Number

When a is a positive real number, the **principal square root of** $-a$ is defined as

$$\sqrt{-a} = i\sqrt{a}$$

Ex. $\sqrt{-3} = \sqrt{3(-1)} = \sqrt{3}\,\sqrt{-1} = i\sqrt{3}$

Multiply and write in standard form

Ex. $\sqrt{-5} * \sqrt{-10}$

$i\sqrt{5}\cdot i\sqrt{10}$

$i^2\sqrt{50}$

50 → 25 · 2, 25 → 5 · 5

$\boxed{-5\sqrt{2}}$

Ex. $\sqrt{-75} * \sqrt{-75}$

$i\sqrt{75}\cdot i\sqrt{75}$

$i^2\sqrt{5625}$

$(-1)75$

-75

Simplify the complex number and then write in standard form.

ex. $(3i)^4$

$3^4 i^4$

$81(1)$

81

ex. $(-i)^6$

1 −1

$i\,i\,i\,i\,i\,i$

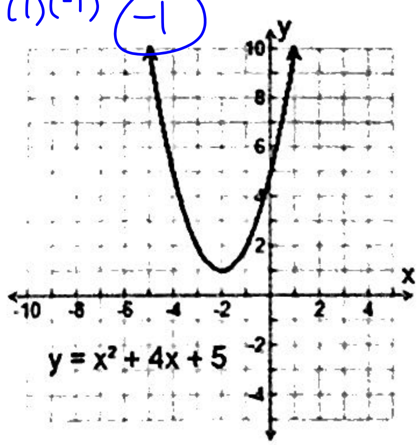
−1

$(-1)^6\, i^6$

$(1)(-1)$

-1

Notice the parabola is not intersecting the x axis.
So the "roots" are not in the ground.
This means the roots are imaginary



$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1

92 → 4 · 23, 4 → 2 · 2

Ex. Find the complex roots:

$8x^2 + 14x + 9 = 0$

$$x = \frac{-14\pm\sqrt{14^2-4(8)(9)}}{2(8)} = \frac{-14\pm\sqrt{196-288}}{16}$$

$$= \frac{-14\pm i\sqrt{92}}{16}$$

$$\frac{-14\pm 2i\sqrt{23}}{16} = \frac{-7\pm i\sqrt{23}}{8} = \boxed{-\frac{7}{8}\pm\frac{\sqrt{23}}{8}i}$$

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