Notes: 2.6 Rational functions

Rational functions:

$$f(x) = \frac{N(x)}{D(x)} \qquad \text{where } D(x) \neq 0$$

Definitions of Vertical and Horizontal Asymptotes

1. The line $x = a$ is a **vertical asymptote** of the graph of f when

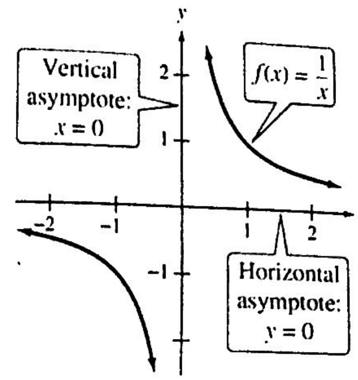
$$f(x) \to \infty \quad \text{or} \quad f(x) \to -\infty$$

as $x \to a$, either from the right or from the left.

2. The line $y = b$ is a **horizontal asymptote** of the graph of f when

$$f(x) \to b$$

as $x \to \infty$ or $x \to -\infty$.



Vertical Asymptotes- find zeros in denominator

$\frac{1}{x} = 0$

Horizontal Asymptotes: Use **degree** of the numerator and denominator "BETC BOTU BOBO"

$\frac{(a)x^2}{(b)x^2}$ $y = \frac{a}{b}$

BETC: num = denom — $y = \frac{a}{b}$ (ratio of the leading coefficients $\frac{num}{den}$)

Bottom = Top

$\frac{x^3}{x^2}$

BOTU: num > denom — none—CHECK FOR SLANT ASYMPTOTE

Bigger on Top

$\frac{x^2}{x^3}$

BOBO: num < denom — y = 0

Bigger on Bottom

$\sqrt{(x-1)^2} = \sqrt{0}$

$x - 1 = 0$

$x = 1$

Examples: find the horizontal and vertical asymptotes and state the domain

Ex. 1 $f(x) = \frac{2x+1}{x+1}$ BETC

$y = \frac{2}{1} = 2$

VA $x + 1 = 0$, $x = -1$

HA: $y = 2$

VA: $x = -1$ D: $\mathbb{R}$, $x \neq -1$

Ex. 2 $\frac{4x^0}{x^2+1}$ BOBO

HA: $y = 0$

VA: None

D: $\mathbb{R}$

VA ~~$x^2 + 1 = 0$~~ ~~$x^2 = -1$~~ ~~$x = \sqrt{-1}$~~ ~~$x = \pm i$~~

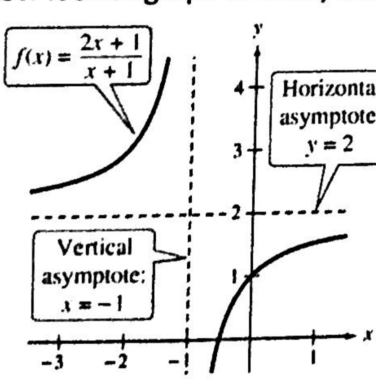
Ex. 3 $\frac{2x^0}{(x-1)^2}$ BOBO

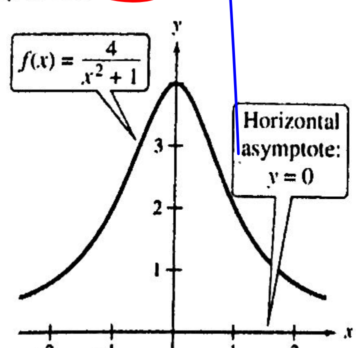
HA: $y = 0$

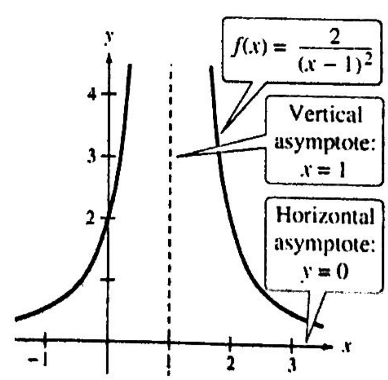
VA: $x = 1$

D: $\mathbb{R}$, $x \neq 1$

Now look at graph to verify the asymptotes:







~~Ex. $f(x) = \frac{5x^2}{x^2 - 1}$~~

Graphing Rational functions:

1. Find the Horizontal Asymptotes: given $f(x)=\dfrac{bx^n+\ldots\ldots}{cx^m+\ldots\ldots}$

n= degree of numerator	b=lead coefficient
m=degree of denominator	c = lead coefficient

a) if $n = m$, then $y = b/c$ is a horizontal asymptote "BETC"

b) asymptote if $n > m$, then there is no horizontal asymptote "BOTU"

c) if $n < m$, then $y = 0$ is a horizontal "BOBO"

* *if no horizontal check for slant asymptotes (num. exactly 1 degree higher then denom.)*

2. Factor the numerator and denominator completely.

3. Find the Points of Discontinuity: a rational graph is discontinuous when the denominator equals 0

There are two types of discontinuity:

a) **Hole in graph (removable):** when a factor in the numerator and denominator are the same. On the graph this will be represented by an open hole in the graph. To find the y-value, substitute "x" back into the simplified function and reduce.

b) **Vertical Asymptote:** when a value of x only makes the denominator equal 0, then the line x = f will be a vertical asymptote.

4. X-intercepts: any factors left in the numerator after simplifying set them equal to zero and these will be the x-intercepts.

5. Y-intercepts: replace all x's in the original problem with 0 and simplify the expression.

Ex. $\frac{3}{x-2}$ BOBO HA: $y=0$

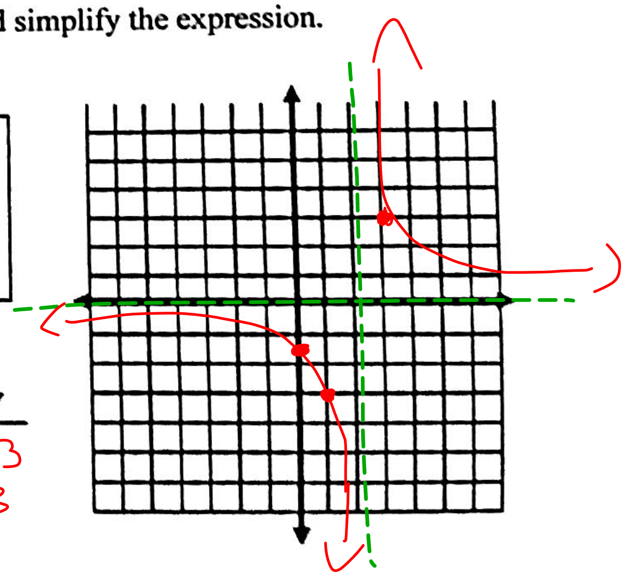
VA
$x-2=0$
$x=2$

D: $\mathbb{R}, x \neq 2$

R: $\mathbb{R}, y \neq 0$

y-int: $(0,-\frac{3}{2})$
x-int: None
vert asymp: $x=2$
horiz asymp: $y=0$

x	y
1	-3
3	3



Ex. $f(x) = \dfrac{x}{x^2-x-2}$ BOBO $y=0$

$x=0$

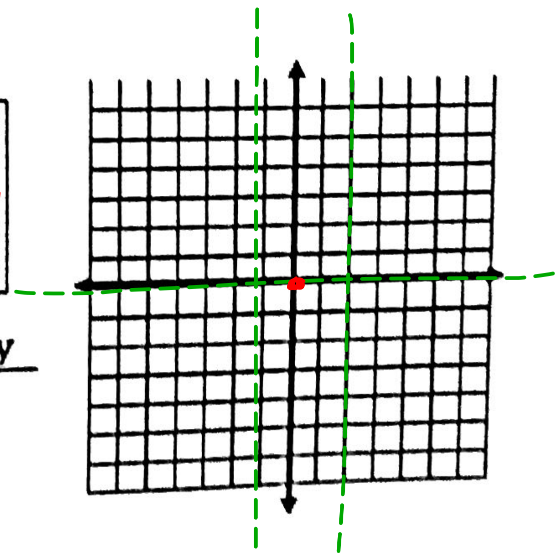
D: $\mathbb{R}, x \neq 2, -1$

R: $\mathbb{R}, y \neq 0$

VA
$x^2-x-2=0$
$(x-2)(x+1)=0$
$x=2,-1$

y-int: $(0,0)$
x-int: $(0,0)$, $x=0$
vert asymp: $x=2, x=-1$
horiz asymp: $y=0$

x	y

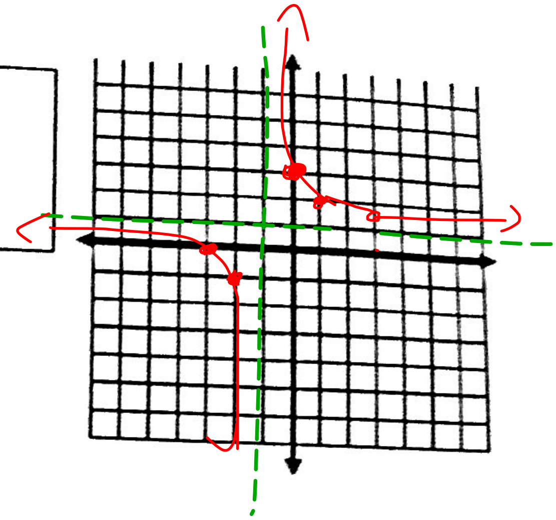


Rational with common factors

Ex. $\frac{x^2-9}{x^2-2x-3}$ BETC $y=1$

HOLE: $(3,\frac{3}{2})$

y-int: $(0,3)$
x-int: $(-3,0)$
vert asymp: $x=-1$
horiz asymp: $y=1$



D: $\mathbb{R},\ x\neq-1$

R: $\mathbb{R},\ y\neq1$

$$\frac{x^2-9}{x^2-2x-3}$$

$$\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x+1)}$$

x	y
1	2
-2	-1

x value at hole

$$\frac{3+3}{3+1}=\frac{6}{4}=\frac{3}{2}\qquad \frac{x+3}{x+1=0}\ VA\qquad \frac{0+3}{0+1}=3\qquad \frac{-2+3}{-2+1}=\frac{1}{-1}=-1$$

If BOTU, then check for a slant:

Slant Asymptotes: If the numerator is <u>exactly</u> one degree higher than the denominator the function has a slant asymptote... USE <u>long division</u> to find the slant asymptote

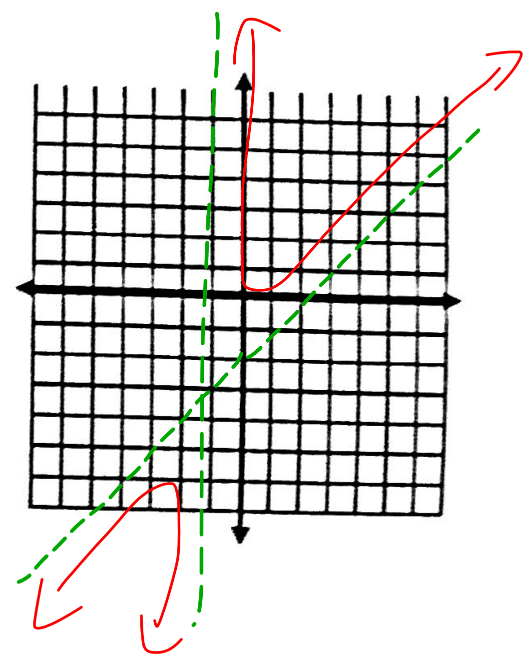
$\frac{0}{1}=0$

Ex. $f(x)=\frac{x^2-x}{x+1}$ BOTU

VA $0=x+1$

$x=-1$

y-int: $(0,0)$
x-int: $(0,0)\ (1,0)$
vert asymp: $x=-1$
~~horiz asymp:~~
slant asymp: $y=x-2$



D: $\mathbb{R},\ x\neq-1$

R: $\mathbb{R}$

$x^2-x=0$

$x(x-1)=0$

$x=0,1$

x	y

$$\begin{array}{r} x-2 \\ x+1\ \overline{)\ x^2-x+0} \\ \underline{x^2\ \ \ +x\ \ \ \ } \\ -2x+0 \\ \underline{-2x-2} \\ 2 \end{array}$$

HW: p. 177 5-8,9,12,13,16,17, 29,30,41,42,43,44,49,54

$x^3 - 4x^2 - x + 4$

$$\frac{\pm 4, \pm 2, \pm 1}{\pm 1} = \pm 4, \pm 2, \pm 1$$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -1 & 4 \\ & \downarrow & 1 & -3 & -4 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array}$$

$$\boxed{x = 1, 4, -1}$$

$x^2 - 3x - 4$

$(x+1)(x-4)$

$x = 4, -1$