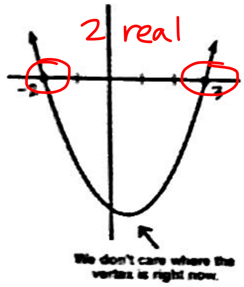
**Notes: 2.5** Zeros of Polynomial Functions

Remember: Complex numbers: a + bi (complex #s include real #s, imaginary #s, or both)

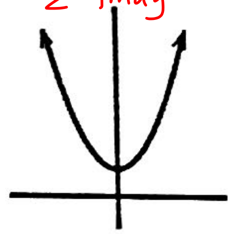
**Conjugate pairs:** complex (imaginary) roots come in pairs. If one imaginary root is given, then the conjugate is also a root.

Find the zeros:

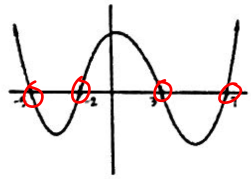
2 real

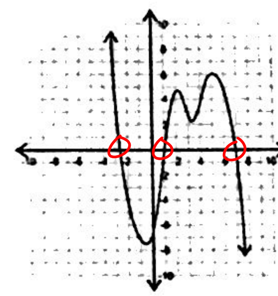



2 imag

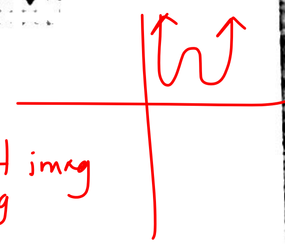


4 real





2 imag
4 turns
3real roots
$x^5$ or $x^7$
$x^9$...



Example: How many zeros? Real? Imaginary?

a) $f(x) = x^1 - 2$

1 zero

b) $f(x) = x^4 - 1$

4 zeros
4 real or 4 imag
2 real / 2 imag

c) $f(x) = x^2 - 6x + 9$

2 zeros
2 real / 2 imag

d) $f(x) = x^5 + x^3 - 1x$

5 zeros
4 imag / 1 real
2 imag / 3 real
5 real

**The rational zero test relates the possible rational zeros of a polynomial (having integer coefficients) to the leading coefficient and the constant term of the polynomial**

**Rational:** a number that can be written as a fraction, a decimal that terminates or repeats, or an integer.

> **The Rational Zero Test**
>
> If the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ has *integer* coefficients, then every rational zero of $f$ has the form
>
> $$\text{Rational zero} = \frac{p}{q}$$
>
> where $p$ and $q$ have no common factors other than 1, and
>
> $p$ = a factor of the constant term $a_0$
>
> $q$ = a factor of the leading coefficient $a_n$.

Ex 1: List the possible zeros $f(x) = 1x^3 - 5x^2 + 2x + 8$

q ... p

$\frac{p}{q} = \frac{\pm 1, \pm 8, \pm 2, \pm 4}{\pm 1} = \pm 1, \pm 8, \pm 2, \pm 4$

p represents the factors of 8,
q represents the factors of 1

Now use the above to find the zeros:

$\cancel{8 \,|\, 1 \quad -5 \quad 2 \quad 8}$
$\cancel{\quad 8 \quad 24}$
$\cancel{1 \quad 3 \quad 26}$

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 2 & 8 \\ & \downarrow & 2 & -6 & -8 \\ \hline 4 & 1 & -3 & -4 & \boxed{0} \\ & \downarrow & 4 & 4 & \\ \hline -1 & 1 & 1 & \boxed{0} & \\ & \downarrow & -1 & & \end{array}$$

*
$x^2 - 3x - 4$
$(x-4)(x+1) = 0$
$x = 4$ & $x = -1$

zeros

Ex. 2 List the possible zeros and use to find the zeros $f(x) = x^3 + 4x^2 - 15x - 18$

$\frac{p}{q} = \frac{\cancel{\pm 1}, \pm 18, \pm 9, \pm 2, \pm 3, \pm 6}{\pm 1}$

$$\begin{array}{c|cccc} 3 & 1 & 4 & -15 & -18 \\ & \downarrow & 3 & 21 & 18 \\ \hline & 1 & 7 & 6 & \boxed{0} \end{array}$$

zeros

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$\boxed{x = -6,\ x = -1,\ x = 3}$

Ex. 3 Find the remaining zeros of the polynomial: $f(x) = x^3 - x^2 + 4x - 4$, given that 1 is a zero.

$x^2(x-1) + 4(x-1)$

$(x-1)(x^2+4) = 0$

$\boxed{x=1}$ $\quad x^2+4=0$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

$\boxed{x = 1, 2i, -2i}$

$\boxed{x = 5 \quad x = -3}$

Ex. 4 Find the remaining zeros of the polynomial: $f(x) = \boxed{x^4 - 2x^3 + 19x^2 - 8x + 60}$, given that 2i is a zero.

$(x-5)(x+3) = 0$

$x^2 - 2x + 15 = 0$

$x^2 + 0x + 4 \,\overline{)\, \cancel{x^4} - 2x^3 + 19x^2 - 8x + 60}$

$- (x^4 + 0x^3 + 4x^2)$

$-2x^3 + 15x^2 - 8x$

$-\cancel{2x^3} + 0x^2 - \cancel{8x}$

$\boxed{2i, -2i}$

$(x-2i)(x+2i)$

$x^2 \cancel{- 2ix + 2ix} - 4i^2$

$\boxed{x^2+4}$

Ex. 5 The following values are zeros/roots/solutions/x-intercepts of a Polynomial; write the polynomial.

-1, 1, 3i, $-3i$

$(x+1)(x-1)(x-3i)(x+3i)$

$(x^2-1)\quad(x^2+9)$

$-9i^2$

$-1$

| | $x^2$ | $-1$ |
|---|---|---|
| $x^2$ | | |
| $9$ | | |

$x^4 + 8x^2 - 9$

Hw p. 164 9-12all, 21,22,30,37,41,45,46,57,78,79